

Extra Questions

1. Evaluate the limit, if exists.

(a) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{x^2 \sec x} \right)$

(b) $\lim_{x \rightarrow \infty} \frac{\ln(2 + e^x)}{3x}$

2. Find the centre, foci and vertices for the following conic sections and sketch the graph.

(a) $9x^2 - 4y^2 - 18x - 16y + 29 = 0$

(b) $25x^2 + 16y^2 + 150x - 128y - 1119 = 0$

3. Evaluate the following integrals

(a) $\int \frac{x \tan^{-1} x}{\sqrt{1+x^2}} dx$

(b) $\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$

(c) $\int \frac{(x-3)^2}{\sqrt{-x^2 + 6x - 5}} dx$

(d) $\int \sqrt{1 + \sin x} dx$

4. Determine if the following improper integral converges or diverges. If it converges, find its value.

(a) $\int_3^{\infty} \frac{dx}{x^2 - 2x - 3}$

(b) $\int_0^1 x \ln x dx$

5. Find the length of the arcs

(a) $x = t - a \tanh \frac{t}{a}, \quad y = a \operatorname{sech} \frac{t}{a} \quad (a \leq t \leq 2a),$

(b) $x = \tan^{-1} t, \quad y = \frac{1}{2} \ln(t^2 + 1) \quad (0 \leq t \leq 1).$